Improved Transmission Probabilities for the Method of Characteristics

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Outline

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The Method of Characteristics

Classical Approach

- Transport equation in a geometric domain D:

$$\begin{cases} (\Omega \cdot \nabla_{\mathbf{r}} + \Sigma)\psi = q, & (\mathbf{r}, \Omega) \in D \times S_N \\ \psi = \psi_0 + \beta\,\psi, & (\mathbf{r}, \Omega) \in \partial D \times S_{N-} \end{cases}$$

- D is composed of unstructured homogeneous regions:

$$\begin{cases} \Sigma(\mathbf{r}) = \Sigma_i, \\ q(\mathbf{r}) = q_i, \end{cases} \qquad \mathbf{r} \in \text{region } i$$

The Method of Characteristics

Classical Approach

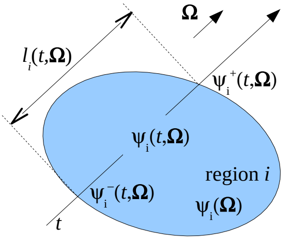
Integrating the Boltzmann Equation over a line segment intersecting a region yields:

- Transmission equation:

$$\psi_i^+(t,\Omega) = \psi_i^-(t,\Omega)\, e^{-\Sigma_i\, l_i(t,\Omega)} + \frac{1-e^{-\Sigma_i\, l_i(t,\Omega)}}{\Sigma_i}\, q_i(\Omega)$$

- Balance equation:

$$\psi_i(t,\Omega) = \frac{q_i(\Omega)}{\Sigma_i} + \frac{\psi_i^-(t,\Omega) - \psi_i^+(t,\Omega)}{\Sigma_i\, l_i(t,\Omega)}$$

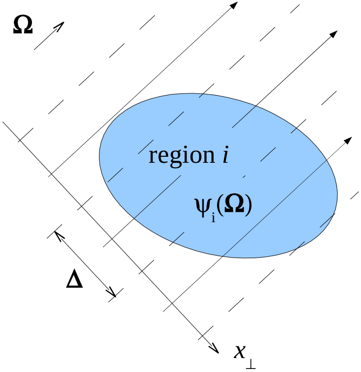


The Method of Characteristics

Classical Approach

- Transverse integration :

$$\psi_i(\Omega) = \frac{\sum_t \Delta\ l_i(t,\Omega)\ \psi_i(t,\Omega)}{\sum_t \Delta\ l_i(t,\Omega)}$$

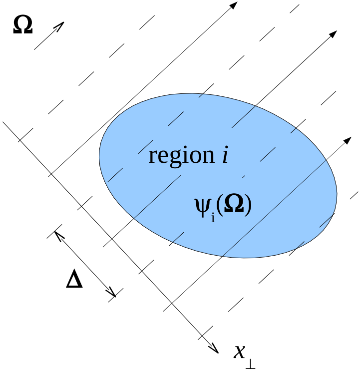


The Method of Characteristics

Tracking and Discretization Problems

Approximations due to the tracking:

- Transverse variation of the angular flux
- Transverse variation of the intersection length
- Material discontinuities

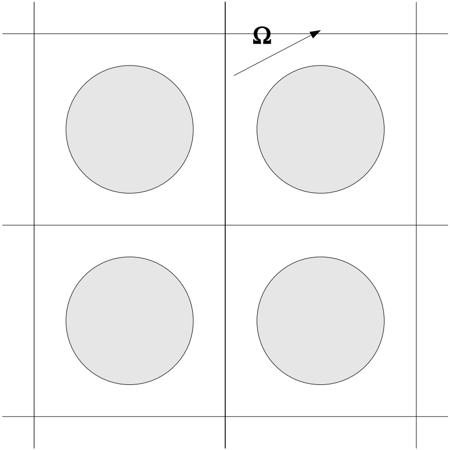


Outline

The Macroband Method

Avoiding Material Discontinuities

Direct projection of all discontinuities:

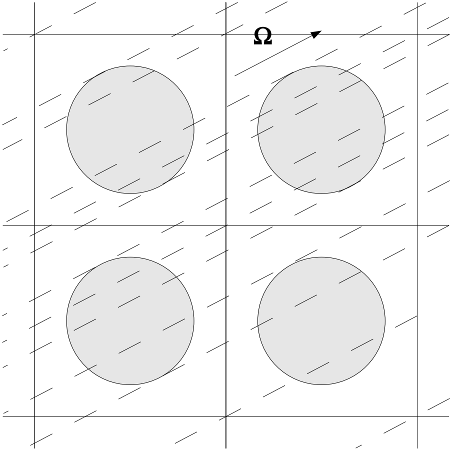


The Macroband Method

Avoiding Material Discontinuities

Direct projection of all discontinuities:

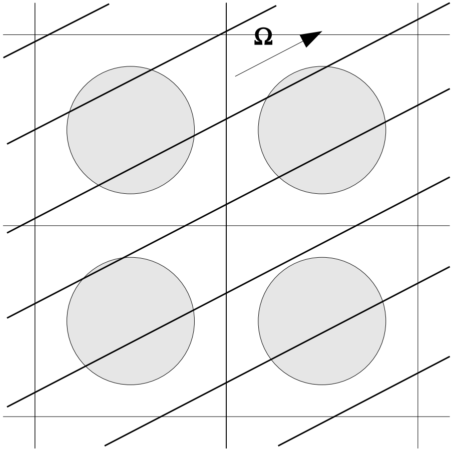
- Large number of transverse mesh cells
- Too onerous



The Macroband Method

Avoiding Material Discontinuities

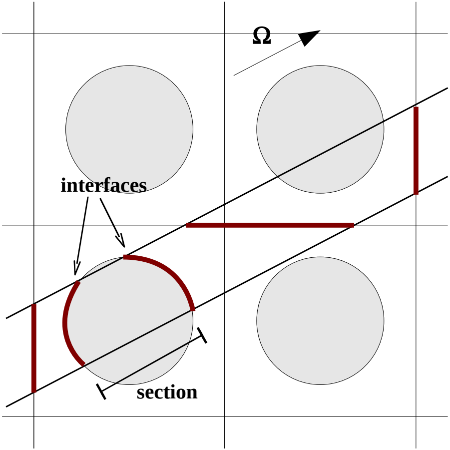
1. Define a constant-step transverse mesh



The Macroband Method

Avoiding Material Discontinuities

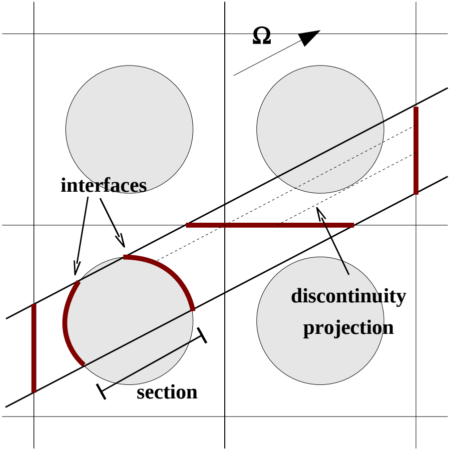
1. Define a constant-step transverse mesh
2. Split each band into sections



The Macroband Method

Avoiding Material Discontinuities

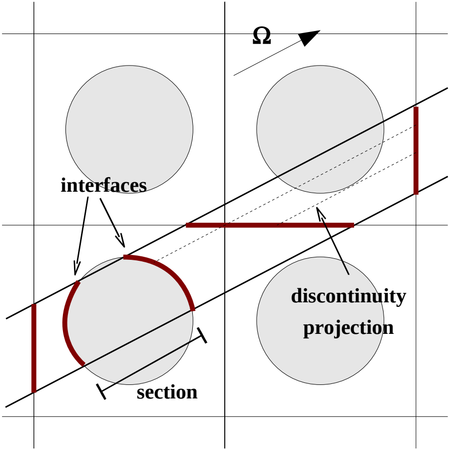
1. Define a constant-step transverse mesh
2. Split each band into sections
3. Project the discontinuities section-wise



The Macroband Method

Avoiding Material Discontinuities

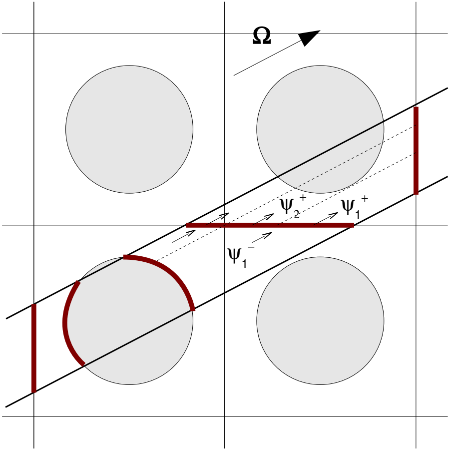
1. Define a constant-step transverse mesh
2. Split each band into sections
3. Project the discontinuities section-wise
4. Propagate the flux across each sub-band



The Macroband Method

Avoiding Material Discontinuities

1. Define a constant-step transverse mesh
2. Split each band into sections
3. Project the discontinuities section-wise
4. Propagate the flux across each sub-band
5. Redistribute the flux at section interfaces

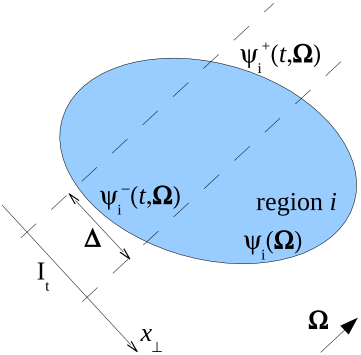


The Macroband Method

Transverse Integration Formula

- Transverse averaged transmission:

$$\psi_i^+(t,\Omega) = e^{-\Sigma_i l_i(t,\Omega)} \psi_i^-(t,\Omega) + \frac{1 - e^{-\Sigma_i l_i(t,\Omega)}}{\Sigma_i} q_i(\Omega)$$



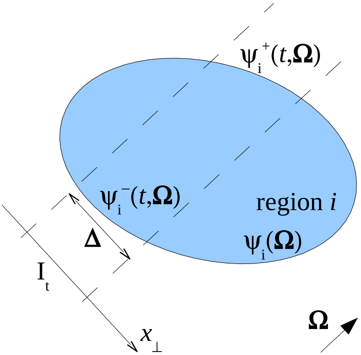
The Macroband Method

Transverse Integration Formula

- Transverse averaged transmission:

$$\psi_i^+(t,\Omega) = T_i(t,\Omega)\,\psi_i^-(t,\Omega) + \frac{1 - T_i(t,\Omega)}{\Sigma_i}\,q_i(\Omega)$$

$$T_i(t,\Omega) = \frac{1}{\Delta}\int_{I_t} e^{-\Sigma_i\, l_i(x_\perp,\Omega)}\,dx_\perp$$

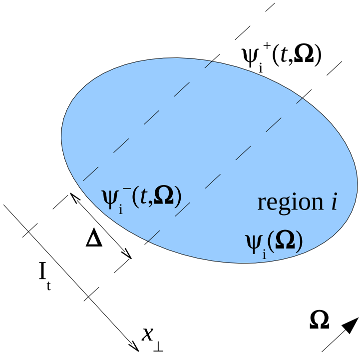


The Macroband Method

Transverse Integration Formula

- Taylor expansion for the exponential term:

$$T_i(t,\Omega) = \frac{1}{\Delta}\int_{I_t} e^{-\Sigma_i\, l_i(x_\perp,\Omega)}\, dx_\perp$$



The Macroband Method

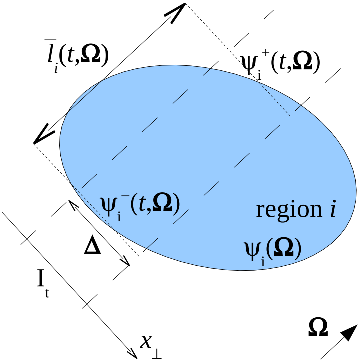
Transverse Integration Formula

- Taylor expansion for the exponential term:

$$T_i(t,\Omega) = \frac{1}{\Delta}\int_{I_t} e^{-\Sigma_i\, l_i(x_\perp,\Omega)}\, dx_\perp$$

$$\simeq e^{-\Sigma_i \bar{l}_i(t,\Omega)} \sum_{p=1}^{n_k} \alpha_p \Sigma_i^p$$

$$\alpha_p = \frac{(-1)^p}{\Delta\, p!}\int_{I_t} \left[l_i(x_\perp) - \bar{l}_i(x_\perp)\right]^p\, dx_\perp$$



Outline

Numerical Results

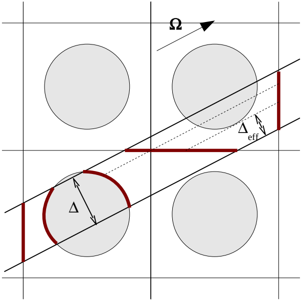
Comparison between the "classical" MOC and the macroband method

- Reference calculation:
  "classical" MOC with $\Delta = 5.10^{-4}$ cm

- Effective tracking step for the macrobands:

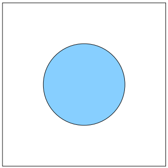
$$\Delta_{eff} = \frac{\Delta}{n_{sb}}$$

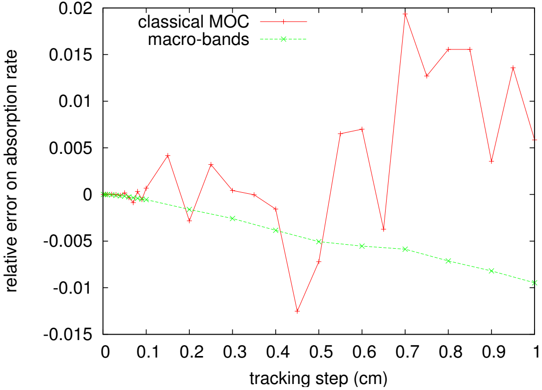
n_{sb} : average number of sub-bands per section.



Numerical Results

Convergence – Accuracy

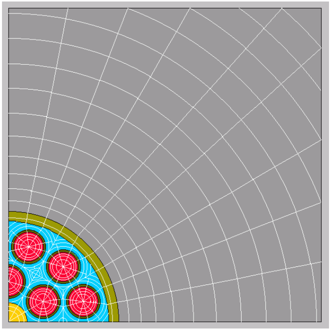


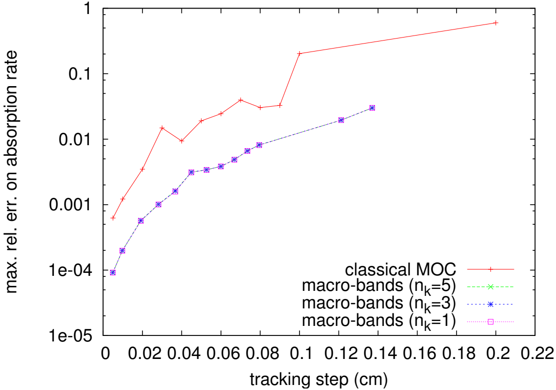


Comparison between the “classical” MOC and the macroband technique
simplified PWR Fuel Cell

Numerical Results

Accuracy

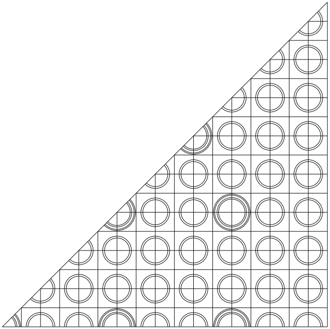


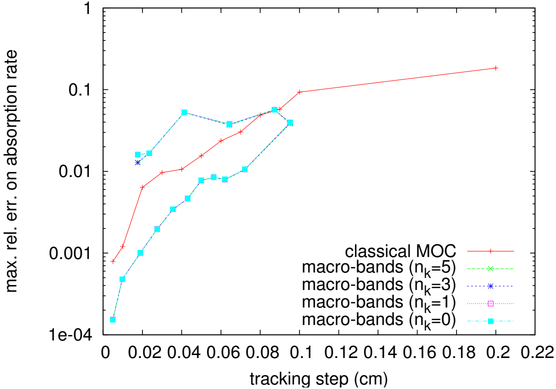


Comparison between the "classical" MOC and the macroband technique
RBMK Cell

Numerical Results

Accuracy





Comparison between the “classical” MOC and the macroband technique
PWR Rodded Assembly

Outline

Conclusions

- Numerical results obtained with the macroband method:
  - for the same precision: tracking step up to $6\times$ larger;
  - for the same number of bands: $2-3\times$ more operations;
  - up to $30-50\%$ gains in terms of computing time.
- Accuracy of the MOC
  - Gain in precision with the macroband method
    $\Rightarrow$ accuracy of the MOC limited by the transverse integration.
  - Negligible impact of the Taylor expansion order (n_k)
    $\Rightarrow$ main cause of error: material discontinuities.

Perspectives

- Optimize the macroband method implementation
- Implement an acceleration scheme for the macroband
- Implement cycling tracking for closed domains

- Use a piecewise linear transverse expansion for the flux

Thank you for your attention

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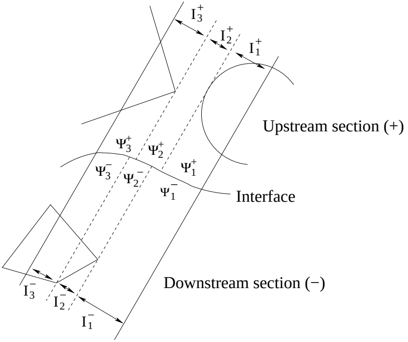
The Macroband Method

Avoiding Material Discontinuities

Flux redistribution at section interfaces:

- Preserve the currents
- Flat flux assumption in each sub-band
- $\psi_k^+ = \sum_{k\prime} \frac{\Delta_{k,k\prime}}{\Delta_k} \psi_{k\prime}^-$

$$\Delta_k = l(I_k^+)$$
$$\Delta_{k,k'} = l(I_k^+ \cap I_{k'}^-)$$



Numerical Results

Computing Time

- Comparison of algorithmic complexities:

tracking technique	exponentials	products	additions	tracking storage
classical MOC	1	2	2	1
Macrobands with Taylor expansion	1	$3 + n_k + r$	$2 + n_k$	$1 + n_k + 2r$
Macrobands only	1	$2 + r$	2	$1 + 2r$

- r: average cost for flux repartition at interfaces

$$r \simeq 0.6$$

Thank you for your attention